Semester One Examination, 2022 Question/Answer booklet

MATHEMATICS

| APPLICATIONS UNIT 3 | | If required by your examination administrator, please place your student identification label in this box |
|---|---------------|---|
| Section One: Calculator-free | | |
| WA student number: | In figures | |
| | Your name | |
| | Your teachers | rs name |
| Time allowed for this se Reading time before commencion Working time: | ng work: five | Number of additional e minutes answer booklets used y minutes (if applicable): |

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet Formula sheet

To be provided by the candidate

pens (blue/black preferred), pencils (including coloured), sharpener, Standard items:

correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is your responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|------------------------------------|-------------------------------|------------------------------------|------------------------------|--------------------|---------------------------|
| Section One: Calculator-free | 7 | 7 | 50 | 51 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 98 | 65 |
| | | | | Total | 100 |

Instructions to candidates

- 1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer booklet preferably using a blue/black pen.
 Do not use erasable or gel pens.
- 3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
- 4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- 5. It is recommended that you do not use pencil, except in diagrams.
- 6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- 7. The Formula sheet is not to be handed in with your Question/Answer booklet.

| Markers use only | | | |
|--------------------|---------|------|--|
| Question | Maximum | Mark | |
| 1 | 5 | | |
| 2 | 5 | | |
| 3 | 7 | | |
| 4 | 10 | | |
| 5 | 8 | | |
| 6 | 7 | | |
| 7 | 9 | | |
| S1 Total | 51 | | |
| S1 Wt (×0.6863) | 35% | | |
| S2 Wt | 65% | | |
| Total | 100% | | |
| | | | |

2

3

Section One: Calculator-free

35% (51 Marks)

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (5 marks)

The statements in parts (a) to (e) of this question relate to graph G shown at right.

For each statement, state whether it is true or false and support your answer with brief reasons.

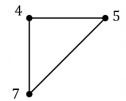


(b) Graph
$$G$$
 is planar. (1 mark)

(c) Graph
$$G$$
 has 2 even vertices. (1 mark)

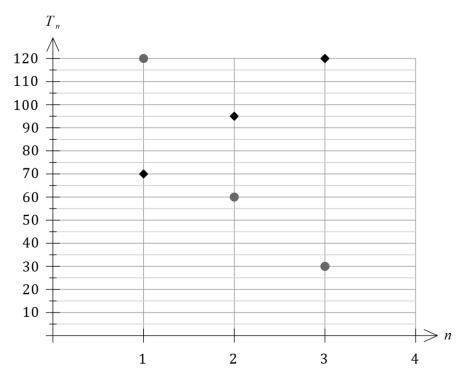
(d) Graph
$$G$$
 has a trail of length S . (1 mark)

(e) Graph F, shown below, is a subgraph of graph G. (1 mark)



Question 2 (5 marks)

The graph below shows the first few terms of an arithmetic sequence and a geometric sequence.



(a) Deduce a rule for the n^{th} term of the geometric sequence.

(2 marks)

(b) Determine the 30th term of the arithmetic sequence.

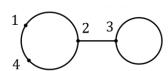
(3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 3 (7 marks)

(a) Graph G_1 is shown at right.

Show that graph G_1 satisfies Euler's formula.



(2 marks)

(b) A haulage company has three trucks and four drivers. The drivers that are licensed to drive each of the trucks are shown with a tick in the following table.

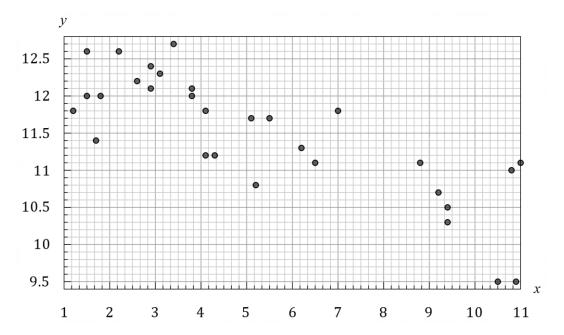
| | | Driver | | | |
|-------|---|--------|---|---|---|
| | | Α | В | С | D |
| | 1 | ✓ | ✓ | | ✓ |
| Truck | 2 | ✓ | ✓ | ✓ | ✓ |
| | 3 | | ✓ | ✓ | ✓ |

(i) Represent the information in the table as bipartite graph G_2 . (3 marks)

(ii) Graph G_2 can be drawn in the plane. Determine, with justification, the number of faces G_2 has. (2 marks)

Question 4 (10 marks)

Over a period of one week, a group of children were observed and the total time that they slept each day recorded. The scatterplot below shows the age in years and months of each child on the x-axis and their average daily sleep time in hours on the y-axis.



The equation of the least-squares line is $\hat{y} = 12.4 - 0.2x$.

(a) How many children aged 2 were in the group?

(1 mark)

(b) Name the explanatory variable.

(1 mark)

(c) Describe the linear association between the variables in terms of direction and strength.

(2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

(d) Interpret the slope of the least-squares line in the context of this question. (2 marks)

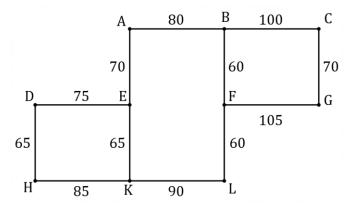
(e) Predict the average daily sleep time of a child who has just had their eighth birthday. (2 marks)

(f) A paediatrician looked at the data and said, " as children age, it causes them to need less sleep". Comment on this statement. (2 marks)

Question 5 (8 marks)

The graph below represents a town centre, with street corners shown as vertices, streets as edges and the length, in metres, of each street as an edge weight.

The total length of all streets in the town centre is 925 m.



(a) List, in order, the vertices that lie on the shortest path from *H* to *C* and state the length of this path in metres. (3 marks)

(b) Does the graph contain an Eulerian trail? Justify your answer. (2 marks)

(c) A cleaning team must make their way along every street in the town centre, starting and finishing at the same street corner. Determine the minimum distance they must walk and explain how they can achieve this minimum. (3 marks)

Question 6 (7 marks)

A complete bipartite graph with n vertices in one group and m vertices in the other group will have nm edges.

(a) Explain the meaning of the term **complete** for such a graph.

(1 mark)

(b) Draw a complete bipartite graph with 5 edges. The graph should clearly be bipartite with its vertices labelled 1, 2, 3, ... and so on. (2 marks)

(c) Construct the adjacency matrix for the graph you drew in part (b). (2 marks)

(d) Determine, with justification, the minimum number of vertices that a complete bipartite graph with 24 edges can have. (2 marks)

| Question 7 (9 m | arks) |
|---|-------|
| A small animal sanctuary has six buildings spread out over an area of land. There are paved | t |
| roads between buildings 1 and 2, 2 and 3, 2 and 5, 3 and 4, 3 and 5, and 5, and 6 | |

roads between buildings 1 and 2, 2 and 3, 2 and 5, 3 and 4, 3 and 5, and 5 and 6.

(a) (2 marks) Construct graph *G* to represent this network of buildings and paved roads.

- (b) Determine the length of
 - (i) the longest trail in graph G.

- (1 mark)
- (ii) the shortest, closed walk in graph *G* that visits all six vertices.
- (1 mark)

- (c) State, with justification, whether graph *G* is a Hamiltonian graph.
- (2 marks)

(d) A new paved road is planned between two different buildings. With the addition of this road, graph G will be a semi-Hamiltonian graph. Determine the number of possible pairs of buildings that the road could be built between. Justify your answer. (3 marks) Supplementary page

Question number: _____